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OCEAN SURFACE REFLECTION LOSS:  
STATUS REPORT AND PREDICTION PROCEDURES.

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## ABSTRACT

### Introduction

1 The best single book on the subject under review ~~is~~ contains some 350 references and is <sup>at this writing,</sup> ~~now~~ over 5 years old. The method adopted in this report is to start with a very brief outline of the theory in that monograph by Beckmann and Spizzichino, specialize to the case of scatter in the specular direction and bring the theory up to date (Section I<sup>1</sup>). Section (II) provides simplified prediction equations and graphs based on the theory of Section I.

3 Section (III) is devoted to a summary of the major known inadequacies of the present prediction scheme and brief statements of problems that must be solved before ~~we can evaluate and improve~~ the accuracy of the current predictions, <sup>can be evaluated and improved</sup>

ABSTRACT

### I Theory

#### A. The Fully Developed Sea

We assume that the sea surface from which the sound is scattered is homogeneous, and that the temporal variation of sea heights at a point or the spatial variation at any instant of time [2] are Gaussian distributed with mean value zero,

$$\langle \zeta \rangle = 0$$

and with variance,  $\sigma^2$ ,

$$\langle \zeta^2 \rangle = \sigma^2$$

The surface height correlation function is [3]

$$C = \frac{Z}{\sigma^2} = \frac{\langle \zeta(0,0,0) \zeta(u,v,\tau) \rangle}{\sigma^2}$$

and the Fourier transform of  $Z$  yields the three-dimensional spectrum of the sea

$$X(\kappa_x, \kappa_y, \Omega) = (2\pi)^{-3} \int_{\tau} \int_{u,v} Z(u,v,\tau) \exp[i(\kappa_x u + \kappa_y v - \Omega \tau)] du dv d\tau$$

where  $u$  = displacement in  $x$  direction between two surface points

$v$  = displacement in  $y$  direction between two surface points

$\tau$  = displacement in time

$\kappa_x, \kappa_y$  = components of surface wave propagation constant

$\Omega$  = angular frequency of surface wave

We shall assume the gravity wave relation between angular frequency, propagation constant and acceleration of gravity,  $g$ :

$$\Omega^2 = \kappa g$$

This permits the spectrum of the sea to be specified in two dimensions either as

$$X = \Psi(\kappa_x, \kappa_y) \delta(\sqrt{g\kappa} - \Omega)$$

or

$$X = \Psi(\kappa, \alpha) \delta(\sqrt{g\kappa} - \Omega)$$

where the Dirac delta function,  $\delta(\sqrt{gk} - \Omega)$ , simply selects the wave that fulfills the gravity wave relation.

The more commonly measured frequency spectrum  $\Phi(\Omega)$  is now obtained by using the energy density coordinate transformation

$$\Psi k dk = \Phi(\Omega) d\Omega .$$

The frequency spectrum is more simply expressed as the integral of the polar form of the  $k$  spectrum over all azimuth angles

$$\Phi(\Omega) = \frac{2\Omega^3}{g^2} \int_0^{2\pi} \Psi(k, \alpha) d\alpha \quad (1)$$

The semi-empirical form of the frequency spectrum for the fully-developed sea that is most widely accepted today (Pierson-Moskowitz) [4] is

$$\Phi(\Omega) = \frac{\alpha g^2}{\Omega^5} \exp \left[ -\beta \left( \frac{\Omega_0}{\Omega} \right)^4 \right] \quad (2)$$

where

$$\alpha = 8.1 \times 10^{-3}$$

$$\beta = 0.74$$

$$\Omega_0 = g/W_{19.5}$$

$$W_{19.5} = \text{Wind speed at 19.5M above sea surface}$$

$$g = \text{acceleration of gravity in units consistent with those of } W .$$

The three parameters of the sea that are most significant in specifying the sound scatter are the mean square surface height,  $\sigma^2$ , the mean square surface slope,  $\Sigma^2$ , and the surface correlation function,  $C$ .

We obtain the mean square height by integrating the frequency spectrum, Eq. (2):

$$\sigma^2 = \int_0^\infty \Phi(\Omega) d\Omega$$

$$\sigma^2 = \frac{\alpha W^4}{4 \beta g^2} \quad (3)$$

For the mean square slope, we will use the relation determined from optical measurements of the sea [5]

$$\Sigma^2 = 5.12 \times 10^{-5} W_{41} + 0.003 \pm 0.004 \quad (4)$$

where  $W_{41}$  is measured in cm/sec at a height of 41 feet over the surface. Unfortunately the measured speed is sensitive to the height of the anemometer.

Finally, for simplicity, and because of lack of sufficient two-dimensional sea data, we will assume that the spatial correlation function is of Gaussian form and isotropic:

$$C(l) = e^{-(l/L)^2} \quad (5)$$

where

$$l = \sqrt{u^2 + v^2}$$

and  $L \equiv$  surface correlation length.

The assumption of an isotropic Gaussian function makes it possible to calculate the correlation length from a knowledge of rms slope  $\Sigma$  and rms

height  $\sigma$  by the relation that is valid for seas of lesser roughness [1]:

$$\Sigma = \sqrt{2} \frac{\sigma}{L} \quad (6)$$

### B. Sound Scattering Theory (Isakovitch-Eckart-Beckmann)

The theoretical solution to the sea surface sound scattering problem usually starts with the Helmholtz Integral:

$$p_Q = \frac{1}{4\pi} \int_S \left[ \frac{e^{ikr_2}}{r_2} \frac{\partial p}{\partial n} - p \frac{\partial}{\partial n} \left( \frac{e^{ikr_2}}{r_2} \right) \right] dS$$

where  $p_Q$  = the scattered acoustic pressure at an interior field point,  $Q$ , of an enclosed volume.

$S$  = integrating surface selected so that acoustic field is zero everywhere except on the insonified water surface.

$k$  =  $2\pi/\lambda$  = acoustic propagation constant

$\lambda$  = acoustic wave length in water

$r_2$  = distance from surface scattering region to field position.

$n$  = normal to surface scattering point.

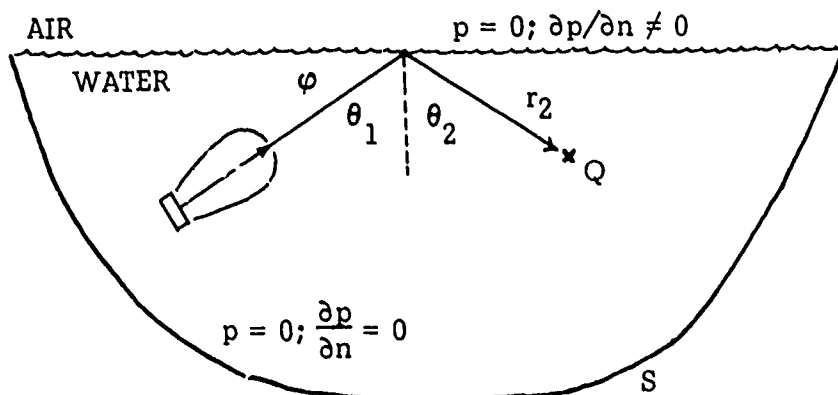


Fig. 1 Scattering Geometry

After applying the boundary conditions for acoustic pressure and acoustic particle velocity at the sea surface, and assuming that there is no shadowing or secondary scattering even for near-grazing incidence, we obtain the integral for the scattered pressure from a given surface of scattering area  $\Delta A$  (now called  $\Delta p_2 \equiv p_Q$ ) in terms of the geometry:

$$\Delta p_2 = \Delta p_{2 \text{ plane}} \frac{F}{\Delta A} \int_{\Delta A} e^{i(\underline{k}_1 - \underline{k}_2) \cdot \underline{r}} dS$$

where  $\Delta p_{2 \text{ plane}}$  = scattered pressure for a plane surface area  $\Delta A$ .

$$= i k p_i \Delta A \cos \theta_1 e^{ik(r_2 - \omega_1 t)/2\pi r_2}$$

$\Delta A$  = insonified surface area over which  $p_i$  and  $\theta_1$  are constant.

$p_i$  = incident sound pressure at sea surface.

$\omega_1$  = incident sound angular frequency

$$F = \frac{1 + \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \theta_3}{\cos \theta_1 (\cos \theta_1 + \cos \theta_2)}$$

$\theta_1$  = angle of incidence (measured with normal)

$\theta_2$  = angle of scatter (measured with normal)

$\theta_3$  = azimuthal angle of scatter plane with respect to incident plane.

$$\underline{k}_1 = k (\underline{i}_1 \sin \theta_1 - \underline{i}_3 \cos \theta_1)$$

$$\underline{k}_2 = k (\underline{i}_1 \sin \theta_2 \cos \theta_3 + \underline{i}_2 \sin \theta_2 \sin \theta_3 + \underline{i}_3 \cos \theta_2)$$

$\underline{i}_1, \underline{i}_2, \underline{i}_3$  are the unit normals in the x, y and z (depth) directions, respectively.



The mean square scattered pressure, which in general includes both coherent and incoherent sound intensity, is formulated:

$$\langle \Delta p_2 \Delta p_2^* \rangle = I_{pl} \frac{F^2}{(\Delta A)^2} \iiint_{x y x' y'} \langle e^{iK_Z [\zeta(x, y) - \zeta(x', y')]} \rangle e^{i[K_x u + K_y v]} dx dx' dy dy'$$

where

$$K = k_1 - k_2$$

$$u = x' - x$$

$$v = y' - y$$

$$I_{pl} = \Delta p_{2 \text{ plane}} \Delta p_{2 \text{ plane}}^*$$

$$* = \text{complex conjugate}$$

$$\langle \rangle = \text{ensemble average}$$

The ensemble average,  $\langle \rangle$ , can be easily evaluated for our surface with its two-dimensional Gaussian distribution:

$$\langle e^{iK_Z (\zeta - \zeta')} \rangle = e^{-[k\sigma (\cos \theta_1 + \cos \theta_2)]^2 [1 - C]}$$

Define the ACOUSTICAL ROUGHNESS OF THE SEA SURFACE  $\equiv R$

$$R \equiv [k\sigma (\cos \theta_1 + \cos \theta_2)]^2 \quad (7)$$

so that the mean scattered intensity can be written

$$\langle \Delta p_2 \Delta p_2^* \rangle = I_{pl} \frac{F^2}{(\Delta A)^2} e^{-R(1-C)} \iiint_{x y x' y'} e^{i(K_x u + K_y v)} dx dx' dy dy'$$

We are interested in the relative scattered intensity, (compared with the plane surface "scattered" intensity) specialized to the requirements of this report, specular scatter, in which  $\theta_1 = \theta_2$ ,  $\theta_3 = 0$  and  $F = 1$ .

$$\langle \rho \rho^* \rangle = \frac{\langle \Delta p_2 \Delta p_2^* \rangle}{I_{\text{plane}}} = e^{-R} + \frac{\pi L^2}{\Delta A} \left( e^{-R} \sum_{m=1}^{\infty} \frac{R^m}{m! m} \right) \quad (8)$$

The two terms of Eq. (8) represent coherent scattered sound (the  $e^{-R}$  term) and incoherent scattering (the summation term). A graph of the term in parentheses (Fig. 2), defined as

$$S(R) \equiv e^{-R} \sum_{m=1}^{\infty} \frac{R^m}{m m!} \quad (9)$$

shows that  $S(R) \rightarrow R$  for  $R \ll 1$

and  $S(R) \rightarrow \frac{1}{R}$  for  $R \gg 1$ .

Since it can be assumed that  $L^2 < \Delta A$ , we can conclude that, in the limits, the specular scatter for a Gaussian sea will be:

$$\text{For Small Acoustical Roughness, } R \ll 1: \langle \rho \rho^* \rangle = e^{-R} \quad (10)$$

$$\text{For Large Acoustical Roughness, } R \gg 1: \langle \rho \rho^* \rangle = \frac{\pi L^2}{R \Delta A} = \frac{\pi}{2 k^2 \Sigma^2 \Delta A \cos^2 \theta_1} \quad (11)$$

It is important to observe that the value of db loss per bounce is an inadequate, and in fact misleading, concept for the rough surface case. In that limit ( $R \gg 1$ ) the specific value of the scattering area becomes

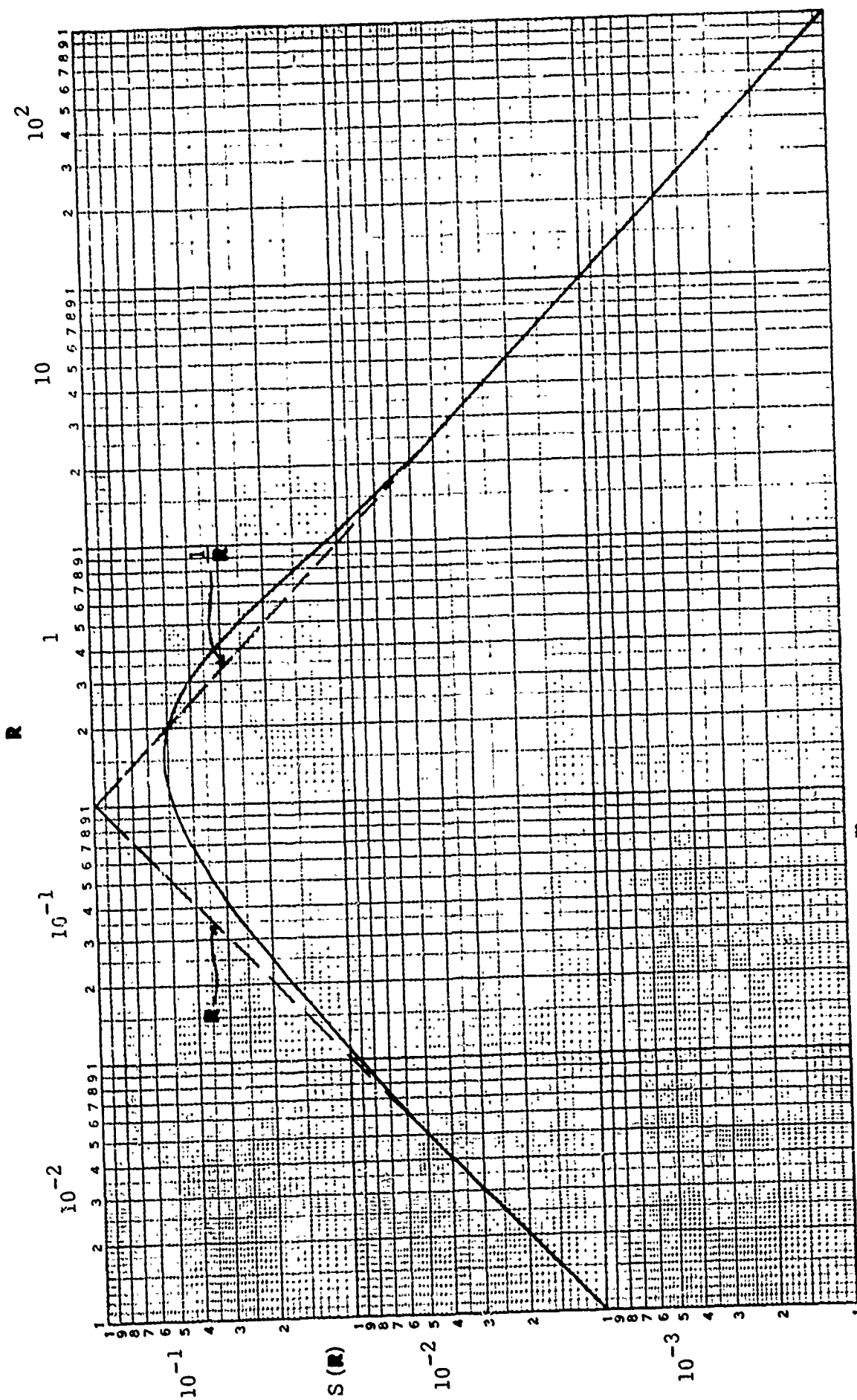


Fig. 2 Graph of  $S(R) = e^{-R} \sum_{m=1}^{\infty} \frac{R^m}{m!}$  as a function of the acoustical roughness  $R$ .

significant and the asymptotic value of  $\langle \rho \rho^* \rangle$  will change from one bounce to the next as the sound wave continues to diverge and scatter.

In addition it should be noted that for an isotropic Gaussian sea the specular scattering loss in the low acoustical roughness limit is dependent on the rms surface height and is completely determined by the value of  $R = 4k^2 \sigma^2 \cos^2 \theta_1$ . On the other hand, in the large acoustical roughness limit, the scattering loss factor will vary inversely as the mean square surface slope (as well as the acoustic propagation constant, the angle of incidence and the scattering area).

### C. Some Other Sound Scattering Theories

The theory outlines in Section IA had its origins in work by Isakovitch (1952) and Eckart (1953). A different approach, based on a generalization of Rayleigh's solution for a corrugated surface was presented by Marsh et al., [7] and criticised as incorrect by Uretsky and others. Rather than review the theory and the objections of its critics, the appendix of this report will point out the inadequacy of predictions from Marsh's theory. It will be seen that Marsh's theory makes no provision for variation of angle of incidence, that it is consonant with the predictions of the development of Section B only for losses of zero db to approximately 3 db after which it diverges wildly from the conclusions of others and from experiment.

Uretsky [8] has presented an exact solution to the related problem of scattering from a pressure-release, sinusoidal corrugated surface. This work has been used in a laboratory model study by Barnard et al., [9] and has been found to give an accurate description of this type of scattering. The applied interest here lies in the expectation that scattering from swell may be similar to scattering from a corrugated surface.

## II Prediction Procedures

### A. The Relative Contributions of Coherent and Incoherent Components

The conclusion of Section IB

$$\langle p p^* \rangle = e^{-R} + \frac{\pi L^2}{\Delta A} S(R) \quad (8)$$

where

$$R = \frac{16\pi^2 \sigma^2}{\lambda^2} \sin^2 \varphi$$

$$\varphi = \text{grazing angle} = 90^\circ - \theta_1$$

$$S(R) = e^{-R} \sum_{m=1}^{\infty} \frac{R^m}{m!} \quad (9)$$

L = surface height correlation length

constitutes the best current means for estimating the relative mean square scattered pressure (compared to a smooth surface) for specular scatter. The known sources in precision and error, due to the assumptions that lead to Eq. (8), are discussed in Section III.

In order to proceed to a simplified prediction procedure, we present in Fig. 3 a "typical" graph of Eq. (8) plotted with  $R$  as independent variable, based on the assumptions that the increase in  $R$  occurs because of increasing frequency, and that there is no change in  $k^2 \Delta A$  (piston-like source),  $\theta_1$ , or sea condition ( $\sigma$ ,  $L$ ,  $\Sigma$ ). For small acoustical roughness the scattering is coherent and decreases exponentially as  $R$  increases. The incoherent scattering term is initially very small, it increases with  $R$  and approaches an asymptotic value determined by the rms slope  $\Sigma$ .

The value of this asymptote depends, importantly, on the insonified area,  $\Delta A$ . For the example plotted we have assumed an insonified



area of size in  $k^2 \Delta A = 4\pi^2 \Delta A / \lambda^2 = 10^4$ . A larger value of the area will, of course, decrease the magnitude of the incoherent component so that the coherent behavior  $e^{-R}$  will continue to dominate to larger values of  $R$ .

#### B. Decibels per Bounce for the Coherent Component Alone

If we assume that the uniformly insonified surface area is very large ( $k^2 \Delta A \gg 10^6$ ) then, almost regardless of the possible sea slope and grazing angle, the simple coherent scattering relation  $\langle \rho \rho^* \rangle = e^{-R}$  will give the whole picture. Fig. 4 shows this behavior, now expressed in db/bounce, calculated from

$$\text{db/bounce} = 10 \log_{10} e^{-R}$$

as a function of grazing angle, wind speed, and rms height of the sea for the frequencies 50, 100, 800, 1200 and 3500 Hz and the grazing angles  $\varphi = 5^\circ, 10^\circ$  and  $20^\circ$ , based on the simplifying assumptions that have been made. The Pierson-Moskowitz [4] spectra for a fully-developed sea have been used to determine  $\sigma$  from the wind speed. The flaws in these assumptions and the task we face before we can improve the prediction accuracy will be discussed briefly in the next section. The limited experimental support for these theories is summarized in the Appendix.

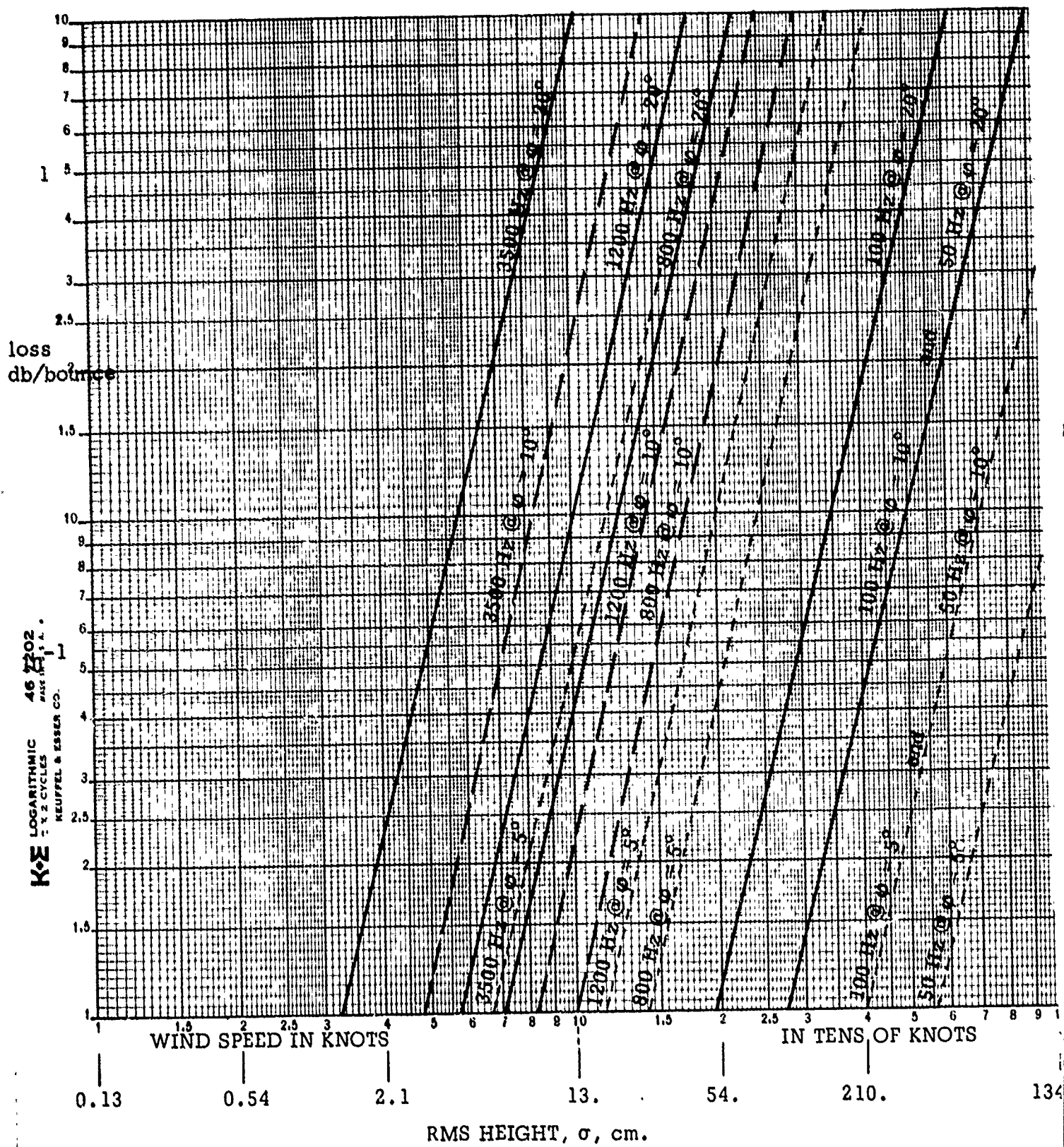


Fig. 4 DB/Bounce for Coherent Scattering from a Pierson-Moskowitz Sea.



### III Future Prediction Procedures, Unsolved Problems

#### A. Surface Shadowing

The theory of Section IB assumes that there is no surface shadowing or secondary scatter from one wave to another. Preliminary attempts to correct for near-grazing shadowing [11] have been based on a ray acoustics approach. It is clear that diffraction into the shadow region must be considered and that the large corrections that Wagner obtains will be decreased for the cases of Navy interest. The fact that the correction should be made when grazing angle is approximately equal to or less than rms slope (e.g.  $\phi \lesssim 6^\circ$  for  $W = 4$  knots,  $\phi \lesssim 13^\circ$  for a 20-knot wind) makes it important that this correction be evaluated as soon as possible. It could not be done within the limits of the present report.

#### B. Near Surface Bubbles

As sound incidence approaches grazing, absorption, dispersion and scattering by near-surface bubbles can become so important that the sound effectively either does not reach the surface or it changes angle of incidence and amplitude in approaching the surface. The number, size, and depth of entrainment of surface-generated bubbles increases with increasing wind speed. The scant literature on the subject includes a laboratory study of bubble generation, one or two in-situ measurements and a few theoretical studies (e.g. ref. [12]). There was inadequate time during the present study to evaluate the effect.

#### C. Non-Fully-Developed Seas

Typically a sea will show different rms slopes in the upwind-downwind direction compared with the cross-wind direction. This will affect the incoherent component (through  $\Sigma$ ) which we have assumed to be negligible for the purpose of simplifying Section IIB.

If we are talking about an attenuated sea, propagated from distant storms ("swell"), the spectrum will no longer be the one described by Pierson and Moskowitz. The high frequencies will have dropped out much more than the lows and the spectrum will be much narrower. Presumably, if the swell spectrum is sufficiently narrow, the behavior will be similar to scattering from a corrugation, whereas if the frequency spread of the swell spectrum is great enough the Gaussian sea approximation would be more accurate. The literature of scattering theory is completely silent on this subject. We still do not know when to solve the problem of scattering from swell by using the solution for a sinusoidal corrugation, or that for a Gaussian sea, or some compromise between these two extremes. The importance of this decision is clear if it is realized that scattering from a corrugation will vary from zero to a maximum as the angle of incidence goes from (a) parallel to an element of the corrugation to (b) in the plane of the sinusoid. On the other hand, for an isotropic Gaussian sea we have seen that the coherent term does not depend on the plane of incidence.

If the sea is not fully-developed, the relation that yields rms height,  $\sigma$ , from wind speed will depend also on the duration of the wind flow at that speed and on whether growth occurred from a white noise background or from a flat calm ocean. It is now possible to estimate this effect [13].

## APPENDIX

### Comparison with Experiment

Verification of the accuracy of scattering predictions from Eq. (8) have been foiled at sea by unknown or poorly defined measurement of the oceanographic parameters. Specifically, use of Eq. (8) requires knowledge of the rms height,  $\sigma$ , the rms slope,  $\Sigma$ , and the area which is uniformly insonified. If the acoustical roughness,  $R$ ,  $\ll 1$ , the only oceanographic parameter required for the prediction of db loss/bounce, is the rms height.

Looking at the history of ocean surface scattering experiments we find:

(A) Marsh [10] shows data from a large number of unpublished ocean experiments in order to compare with his theory

$$\text{db/bounce} = -10 \log [1 - 0.0234b^{3/2}]$$

$$b = (F, \text{k cps}) (\text{Wave Height, } H, \text{ in Ft.})$$

Marsh assumes that there is no explicit dependence on angle of incidence. However, if a Gaussian distribution is assumed, we can take  $H = 6\sigma$  (to include 99% of the values of height), and convert Marsh's formula to

$$\frac{\text{db}}{\text{bounce}} = -10 \log_{10} \left[ 1 - (.0234) \left( \frac{\sqrt{R}}{.42 \cos \theta_1} \right)^{3/2} \right]$$

Unfortunately, no angles of incidence or sea conditions are given for Marsh's data which fits his theory reasonably well as long as db/bounce  $< 3.0$ .

Marsh's data for  $R < 1$  also fits quite well to Eq. (8) if we make the reasonable assumption  $\theta_1 = 80^\circ$  ( $10^\circ$  grazing).

The theory developed by Marsh has been rejected in this report because: it is valid only for  $R < 1$ ; it contains no explicit provision for introduction of dependence on angle of sound incidence.

(B) Project AMOS [14] includes an estimate of db loss/bounce given in terms of depth of the isothermal layer which can be restated as

$$\text{db/bounce} = 2.3\sqrt{f} \quad \text{for sea state} < 3$$

$$\text{db/bounce} = 4.6\sqrt{f} \quad \text{for sea state} \geq 3$$

where  $f$  = sound frequency in kHz.

These formulas are too crude and too divergent from recent studies to be considered further. The data may have been affected by bubbles.

(C) Proud, Beyer and Tamarkin [15] performed a laboratory study, using a one-dimensional cork model of a Gaussian Sea, and showed that the theory, such as presented in Section IB, is essentially correct, and that forward scattering depends only on acoustical roughness

$$R = \frac{16\pi^2 \sigma^2 \cos^2 \theta_1}{\lambda^2}$$

slope  $\Sigma$  and area insonified for angles of incidence from  $0^\circ$  to  $60^\circ$ , for  $0.3 < R < 2$ .

(D) Medwin [6] used a wind-driven, scaled, model-sea surface of a large tank of water to study mean value and probability distribution of normally incident, forward-scattered acoustic pressure over the range  $0.25 < R < 25$ . The oceanographic parameters, rms height, height distribution, rms slope and its distribution in the up-down wind direction and cross-wind direction

were measured directly in order to check the predictions of theory. Accurate predictions were obtained when Eq. (8) was used.

(E) Scheible et al., [16] have used the same laboratory set-up as in (D) to study near-grazing forward scatter in the case  $R < 0.3$  where only the coherent scattering term should be important. Their work suggests that near-grazing scattering lies between the value given by the coherent term of Eq. (8) and the ray-corrected expression due to Wagner [11].

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